Fast Hue and Range Preserving Histogram Specification:
Theory and New Algorithms for
Color Image Enhancement

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1. Introduction

Objective: color image enhancement methods satisfying:

- hue preservation

  dominant color ingredient;

  invariant under changes of direction and intensity of incident light

- optimal range (gamut) preservation

  many enhancement algorithms yield pixel values beyond their range which are clipped back to the boundaries (e.g., $[0, 255]$ for 8-bit images);

- low computational complexity

  mega-pixel images, limited resources in hardware implementations, video.



Left: Original (matlab IPT credits). Middle: HE of R,G,B independently. Right: HE of the intensity channel in HSI – 36.1 % of the pixels have values in $(255, 443.5]$.

◇ Our approach is based on histogram specification

◇ Other methods are PDE-based and variational
automatic color enhancement (ACE) of Rizzi/Gatta/Marini 2003, Bertalmio/Caselles/Provenzi/Rizzi 2007, Getreuer (IPOL implementation 2012) Recent method of Morel/Petro/Sbert via screened Poisson equation (2013).

◇ We look for flexible (not fully automatic) enhancement algorithms:
  – "The chemical compounds that form color receptors vary among the population. The physical shapes of the receptors vary among the population and within the retina. Thus, the color vision among observers varies significantly." [Berns 2000]
  – Image improvement/enhancement is always driven by an application


2.Preliminaries

$w = (w_r, w_g, w_b)$ – $M \times N$ RGB image, $w_c \in \{0, \ldots, L-1\}$, $c \in \{r, g, b\}$

each w_c is stored as an $n := MN$ vector / $\mathbb{I}_n := \{1, \cdots, n\}$

A. Histogram Specification (HS) and Strict Ordering

– The intensity of w is $f(w) := \frac{1}{3}(w_r + w_g + w_b)$.

– f has $K = 3(L-1)+1$ different values   8-bit image $\Rightarrow$ $L = 256$

– HS: we want to convert f into $\widehat{f}$ with $\widehat{f}[i] \in \{0, \ldots, L-1\}$ having a specified (target) histogram $\widehat{h} = (\widehat{h}_1, \ldots, \widehat{h}_L)$, i.e., $\widehat{h}[k] := \sharp\{i \in \mathbb{I}_n : \widehat{f}[i] = k-1\}$, $k = 1, \ldots, L$

order the pixels in f: $i \prec j$ if $f[i] < f[j]$

$$\underbrace{i_1 \prec i_2 \prec \cdots \prec i_{\zeta[1]}}_{\widehat{h}[1]} \prec \cdots \prec \underbrace{i_{n-\zeta[L]+1} \prec \cdots \prec i_n}_{\widehat{h}[L]}$$

– Ill-posed problem for digital (quantized) images since $n \gg K$

– An issue: obtain a meaningful total strict ordering of all pixels in f

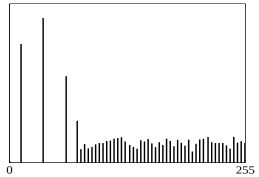
Histogram equalization (HE) is a particular case of HS where $\widehat{h}[k] = n/L \quad \forall\, k \in \mathbb{I}_L$

Histogram Equalization

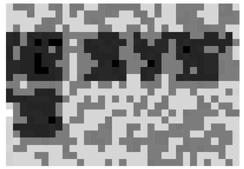
original

(a) histeq

histogram of (a)



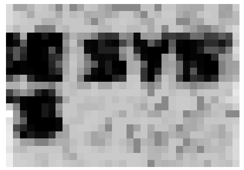
zoom of (a)



(b) Algorithm 1

histogram of (b)

zoom of (b)



Contemporary sorting algorithms

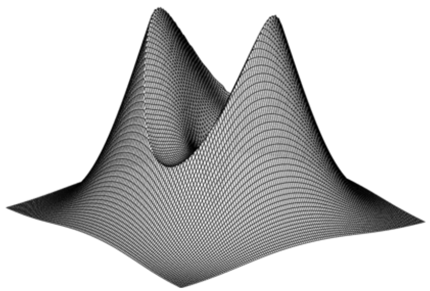
- Feature vector algorithms:
  - Local Mean Algorithm (LM) [Coltuc, Bolon, Chassery 2006]
  - Wavelet Approach (WA) [Wan, Shi 2007]

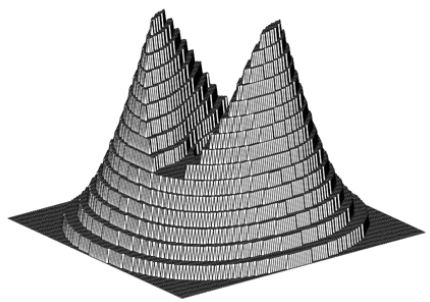
  Drawbacks: storage demanding (LM: 6 images, WA: 9 images) and ineffective ordering.

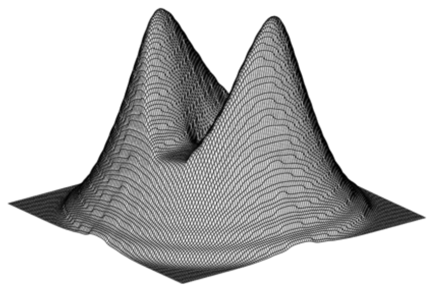
- **Variational approach: sort $\widehat{u}$** [Nikolova, Wen, R.Chan 2012]

$$\widehat{u} = \arg\min_u J(u,f) := \sum_{i\in\mathbb{I}_n} \psi(u[i]-f[i]) + \beta \sum_{i\in\mathbb{I}_r} \varphi((\nabla u)[i]), \quad \beta > 0$$

ψ and φ – smooth approximations of $t \mapsto |t|$ ($J(\cdot,f)$ – a fully smoothed ℓ_1–TV model)


Real-valued original


f quantized on $\{0,\cdots,15\}$


Restored $\widehat{f}$

We choose $\psi = \varphi = |t| - \alpha \log\left(1 + \dfrac{|t|}{\alpha}\right)$

Algorithm 1: Fast HS using Strict Ordering [Nikolova, Steidl 2014]

Initialization: $u^{(0)} = f$, $\alpha := \alpha_1 = \alpha_2 = 0.05$ and $\beta = 0.1$. $R = 5$ (default)

1. For $r = 1, \cdots, R$, compute

$$u(r+1) = f - \xi(\beta \nabla^T \varphi'(\nabla u)) \quad \text{where} \quad \xi(y) = \frac{\alpha y}{1-|y|} \text{ and } \varphi'(t) = \frac{t}{\alpha + |t|}$$

2. Obtain the ordering $\{i_j\}_{j=1}^n$ of $\mathbb{I}_n$ from the ascending sort of the entries of $u^{(K)}$.

3. For $k = 0, \ldots, L-1$ set $c_{k+1} := c_k + h_k$ and $\widehat{f}[c_k + 1] = \ldots = \widehat{f}[c_{k+1}] = k$.

Facts

– Strict ordering information: There exists a dense open subset $\mathbb{K}^n$ of $\mathbb{R}^n$ so that for any $f \in \mathbb{K}^n$ the minimizer $\widehat{u}$ of $J(\cdot, f)$ obeys $\widehat{u}[i] \neq \widehat{u}[j] \quad \forall\, i, j \in \mathbb{I}_n, \quad i \neq j$ [Nikolova, Wen, Chan 2012]

– $\beta < \frac{1}{4} \quad \Rightarrow \quad \left\{ \begin{array}{l} \|\widehat{u} - f\|_\infty \leq \xi\big(4\beta, \alpha_1\big) \\ \|\widehat{u} - f\|_\infty \nearrow \xi\big(4\beta, \alpha_1\big) \text{ as } \alpha_2 \searrow 0 \end{array} \right.$ [Bauss, Nikolova, Steidl 2013] $\Rightarrow$

Algorithm 1: $\|\widehat{u} - f\|_\infty \leq 0.0333$

– For $R > 6$ the ordering remans almost the same and the upper bound is respected.

– High speed: φ' and ξ are explicit, simple to compute and we know the right initialization.

B. Hue and Range Preservation

$w = (w_r, w_g, w_b)$ is an RGB image with $w_c \in \{0, \ldots, L-1\} \quad \forall\, c \in \{r,\, g,\, b\}$

- Range preservation is a mandatory. A transformed version $\widehat{w}$ of w can be correctly depicted only if $\widehat{w}_c[i] \in [0, L-1] \quad \forall\, i \in \mathbb{I}_n \quad \forall\, c \in \{r,\, g,\, b\}$

  Otherwise, the obtained image is modified according to the visualization device

- Hue: $H(w) = 0$ if $w_r = w_g = w_b$ and otherwise (artistic description of colors)

$$H(w) := \begin{cases} \theta & \text{if } w_b \le w_g, \\ 360 - \theta & \text{if } w_b > w_g, \end{cases}$$

  where

$$\theta := \arccos \frac{\frac{1}{2}((w_r - w_g) + (w_r - w_b))}{\left(\frac{1}{2}((w_r - w_g)^2 + (w_r - w_b)^2 + (w_g - w_b)^2)\right)^{\frac{1}{2}}}$$

  Hue: describes the dominant color ingredient that we perceive. We want to keep them.

  Preserving the hue and enhancing the brightness, the image will appear more colorful.

- The simplest hue and range preserving method is stretching of w but limited improvement

Hue and Range Preserving Models

Objectives: Given an input RGB image w and a target histogram fitted intensity image $\widehat{f}$, we want to find $\widehat{w}$ satisfying

(a) Intensity fit: $\widehat{f} = \frac{1}{3}(\widehat{w}_r + \widehat{w}_g + \widehat{w}_b)$.

(b) Hue preservation: $H(\widehat{w}) = H(w)$;

(c) Optimal range preservation: $0 \leq \widehat{w}_c \leq L-1 \quad c \in \{r, g, b\}$.

Hue preservation: it is sufficient to preserve the hue angle $\frac{\frac{1}{2}((w_r - w_g) + (w_r - w_b))}{(\frac{1}{2}((w_r - w_g)^2 + (w_r - w_b)^2 + (w_g - w_b)^2))^{\frac{1}{2}}}$

Affine transforms (the same for each color pixel) preserve the hue angle:

$$\hat{w}_c[i] = a[i] w_c[i] + b[i], \quad \forall c \in \{r,\ g,\ b\},$$

other hue preserving transforms ?

- $a[i] = 0$ additive transform – **shifting**;
- $b[i] = 0$ linear/multiplicative transform – **scaling**;

  [Naik, Murthy 2003] also shows a way to preserve the range.

Scaling: $\widehat{w}_c[i] = a[i]w_c[i] \quad \forall c \in \{r, g, b\}$

By (a) (intensity fit) $\widehat{f}[i] = \frac{1}{3}(\widehat{w}_r[i] + \widehat{w}_g[i] + \widehat{w}_b[i]) = a[i]\frac{1}{3}(w_r[i] + w_g[i] + w_b[i]) = a[i]f[i]$

$$\Rightarrow \quad a[i] = \frac{\widehat{f}[i]}{f[i]}.$$

Scaling does not preserve the range - $a[i]$ can be very large. Apply RGB to CMY idea:

Algorithm 3 (Naik and Murthy 2003)

1. Compute the intensity f of w and the target intensity $\widehat{f}$.
2. For $i \in \mathbb{I}_n$ compute $c \in \{r, g, b\}$

$$\begin{array}{lll} \text{(i)} & \widehat{w}_c[i] := \frac{\widehat{f}[i]}{f[i]}\, w_c[i] & \text{if} \quad \widehat{f}[i] \leq f[i] \\ \text{(ii)} & \widehat{w}_c[i] := L - 1 - \frac{L-1-\widehat{f}[i]}{L-1-f[i]}(L - 1 - w_c[i]) & \text{if} \quad \widehat{f}[i] > f[i]. \end{array}$$

- The NM algorithm is the state of the art method for gamut correction; (Bassiou/Kotropoulos 2007, Han et al. 2011, Singh/Rewat 2013)
- Drawback: Saturation decreases for any pixel obeying $f[i] < \widehat{f}[i]$.

  The range correction in step 2 of the NM algorithm is not optimal.

3. New Affine Histogram Specification Models

$$\widehat{w}_c[i] = a[i] w_c[i] + b[i], \quad c \in \{r, g, b\}.$$

Summing up show that intensity fit $\widehat{f}[i] = f[i]$ holds if and only if

$$\widehat{w}_c[i] = a[i]\,(w_c[i] - f[i]) + \widehat{f}[i], \quad c \in \{r, g, b\}.$$

We have to adapt the model so that the range is preserved.

$$\begin{aligned} M[i] &:= \max\{w_c[i] : c \in \{r, g, b\}\}, \\ m[i] &:= \min\{w_c[i] : c \in \{r, g, b\}\} \end{aligned}$$

One has $0 \le m[i] \le f[i] \le M[i] \le L - 1.$

Further $M[i] = f[i]$, resp., $m[i] = f[i]$ if and only if $w_r[i] = w_g[i] = w_b[i]$, i.e., $w[i]$ is gray.

- Upper gamut problem if and only if $G_M[i] := a[i](M[i] - f[i]) + \widehat{f}[i] > L - 1$
- Lower gamut problem if and only if $G_m[i] := a[i](m[i] - f[i]) + \widehat{f}[i] < 0$

When a pixel $\widehat{w}[i]$ has a gamut problem, the optimal correction is the *nearest* color value that belongs to the range, having the same intensity and preserving the hue.

Optimal corrections

- Upper gamut: $L - 1 = a[i](M[i] - f[i]) + \widehat{f}[i] \quad \Rightarrow \quad a[i] = \frac{L-1-\widehat{f}[i]}{M[i]-f[i]} \quad \Rightarrow$

$$\widehat{w}_c[i] = \frac{L - 1 - \widehat{f}[i]}{M[i] - f[i]}(w_c[i] - f[i]) + \widehat{f}[i], \quad c \in \{r, g, b\}.$$

- Lower gamut: $0 = a[i]\ (m[i] - f[i]) + \widehat{f}[i] \quad \Rightarrow \quad a[i] = \frac{\widehat{f}[i]}{f[i]-m[i]} \quad \Rightarrow$

$$\widehat{w}_c[i] = \frac{\widehat{f}[i]}{f[i] - m[i]}(w_c[i] - f[i]) + \widehat{f}[i], \quad c \in \{r, g, b\}.$$

A. Affine Algorithm with Optimal Range Preservation

For $a[i]$ we consider a convex combination of the shifting and scaling models:

$$a[i] := \lambda \frac{\widehat{f}[i]}{f[i]} + (1 - \lambda).$$

Affine model:

$$\widehat{w}_c[i] = a[i](w_c[i] - f[i]) + \widehat{f}[i], \quad c \in \{r,\, g,\, b\}.$$

Propositions 1 and 2. Apply the affine model with our Optimal corrections.

- Our upper gamut correction does not introduce a lower gamut problem;
- Our lower gamut correction does not introduce an upper gamut problem.

Algorithm 3 (Optimal Affine Range-Preserving Enhancement)

1. Compute the intensity f of w and the target intensity $\widehat{f}$ using Alg. 1
2. For $i \in \mathbb{I}_n$ compute $M[i]$ and $m[i]$.

2.1 If $f[i] \in \{M[i], m[i]\}$, then $\widehat{w}_c[i] = \widehat{f}[i]$, $c \in \{r,\, g,\, b\}$.

2.2 If $f[i] \neq 0$, compute

$$a[i] := \lambda \, \frac{\widehat{f}[i]}{f[i]} + (1-\lambda) \quad \text{and} \quad \begin{cases} G_m^\lambda := a[i](m[i] - f[i]) + \widehat{f}[i], \\ G_M^\lambda := a[i](M[i] - f[i]) + \widehat{f}[i], \end{cases}$$

and for all $c \in \{r,\, g,\, b\}$

(i) $\widehat{w}_c[i] := a[i]\,(w_c[i] - f[i]) + \widehat{f}[i]$ if $G_m^\lambda[i] \geq 0 \text{ and } G_M^\lambda[i] \leq L-1$,

(ii) $\widehat{w}_c[i] := \frac{L-1-\widehat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \widehat{f}[i]$ if $G_M^\lambda[i] > L-1$,

(iii) $\widehat{w}_c[i] := \frac{\widehat{f}[i]}{f[i]-m[i]}(w_c[i] - f[i]) + \widehat{f}[i]$ if $G_m^\lambda[i] < 0$.

Algorithm 4 (Multiplicative Color Enhancement) $\lambda = 1$

2.2 If $f[i] \neq 0$, compute for all $c \in \{r, g, b\}$:

(i) $\widehat{w}_c[i] := \frac{\widehat{f}[i]}{f[i]} w_c[i]$ if $\widehat{f}[i] \frac{M[i]}{L-1} \leq f[i]$,

(ii) $\widehat{w}_c[i] := \frac{L-1-\widehat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \widehat{f}[i]$ if $\widehat{f}[i] \frac{M[i]}{L-1} > f[i]$.

Algorithm 5 (Additive Color Enhancement) $\lambda = 0$

2.2 If $f[i] \neq 0$, compute for all $c \in \{r, g, b\}$:

(i) $\widehat{w}_c[i] := w_c[i] - f[i] + \widehat{f}[i]$ if $-m[i] \leq \widehat{f}[i] - f[i] \leq L - 1 - M[i]$,

(ii) $\widehat{w}_c[i] := \frac{L-1-\widehat{f}[i]}{M[i]-f[i]}(w_c[i] - f[i]) + \widehat{f}[i]$ if $\widehat{f}[i] - f[i] > L - 1 - M[i]$

(iii) $\widehat{w}_c[i] := \frac{\widehat{f}[i]}{f[i]-m[i]}(w_c[i] - f[i]) + \widehat{f}[i]$ if $\widehat{f}[i] - f[i] < -m[i]$.

$\widehat{w}^{\times}$ by the Multiplicative Algorithm

$\widehat{w}^{+}$ by the Additive Algorithm

For $\lambda \in [0, 1]$ fixed, set

$$\widetilde{w}_c := \lambda \widehat{w}_c^{\times} + (1-\lambda)\widehat{w}_c^{+} \quad \forall\, c \in \{r,\, g,\, b\}.$$

We want to know if this convex combination can replace the affine algorithm.

The sets below describe the pixels corresponding to the upper gamut correction step (ii) and the lower gamut correction step (iii) in our Optimal Affine Algorithm, respectively:

$$\mathcal{U}(\lambda) := \{i \in \mathbb{I}_n : G_M^\lambda[i] > L-1\} \quad \text{and} \quad \mathcal{L}(\lambda) := \{i \in \mathbb{I}_n : G_m^\lambda[i] < 0\}.$$

Proposition 3 One has $\mathcal{L}(1) = \varnothing$ and

$$\mathcal{U}(\lambda_1) \subseteq \mathcal{U}(\lambda_2), \quad \mathcal{L}(\lambda_1) \supseteq \mathcal{L}(\lambda_2), \quad 0 \le \lambda_1 < \lambda_2 \le 1.$$

$$\mathcal{U}(1) = \left\{i : \widehat{f}[i]\, \tfrac{M[i]}{L-1} > f[i]\right\},\ \mathcal{U}(0) = \left\{i : \widehat{f}[i] + M[i] - (L-1) > f[i]\right\},\ \mathcal{L}(0) = \left\{i : \widehat{f}[i] + m[i] < f[i]\right\}.$$

Proposition 4 For the same λ, let $\widehat{w}$ be obtained by the Optimal Affine Algorithm. Then

$$i \in \mathbb{I}_n \setminus \left\{\mathcal{U}(1) \setminus \mathcal{U}(0) \cup \mathcal{L}(0)\right\} \quad \Rightarrow \quad \widetilde{w}_c[i] = \widehat{w}_c[i].$$

nearly all pixels are in this set !!!

Enhancement of the image "couple" by our Optimal Affine Algorithm with $\lambda = 0,\ \frac{1}{4},\ \frac{1}{2},\ \frac{3}{4},\ 1$.

4. Comparison of the Models

$\widehat{w}^{\times}$ – by our Multiplicative ($\times$) Algorithm;

$\widehat{w}^{+}$ – by our Additive ($+$) Algorithm;

$\widehat{w}^{\bullet}$ – by the Naik-Murthy (NM) Algorithm

In the HSI color model, Saturation:

$$S(w) := \begin{cases} 1 - \frac{\min\{w_r, w_g, w_b\}}{f(w)} & \text{if } f(w) > 0, \\ 0 & \text{if } f(w) = 0 \ . \end{cases}$$

A. Saturation Properties

Propositions 5, 6

– Multiplicative Algorithm

$$\begin{array}{lll} \text{(i)} & S(\widehat{w}^{\times}[i]) = S(w[i]) & \text{if} \quad i \in \mathbb{I}_n \setminus \mathcal{U}(1), \\ \text{(ii)} & S(\widehat{w}^{\times}[i]) = S(w[i]) \, \frac{f[i]}{\widehat{f}[i]} \, \frac{L-1-\widehat{f}[i]}{M[i]-f[i]} & \text{if} \quad i \in \mathcal{U}(1). \end{array}$$

– Additive Algorithm

$$\begin{array}{lll} \text{(i)} & S(\widehat{w}^{+}[i]) = S(w[i]) \, \frac{f[i]}{\widehat{f}[i]} & \text{if} \quad i \in \mathbb{I}_n \setminus (\mathcal{U}(0) \cup \mathcal{L}(0)), \\ \text{(ii)} & S(\widehat{w}^{+}[i]) = S(\widehat{w}^{\times}[i]) & \text{if} \quad i \in \mathcal{U}(0), \\ \text{(iii)} & S(\widehat{w}^{+}[i]) = 1 & \text{if} \quad i \in \mathcal{L}(0). \end{array}$$

– Naik-Murthy (NM) Algorithm

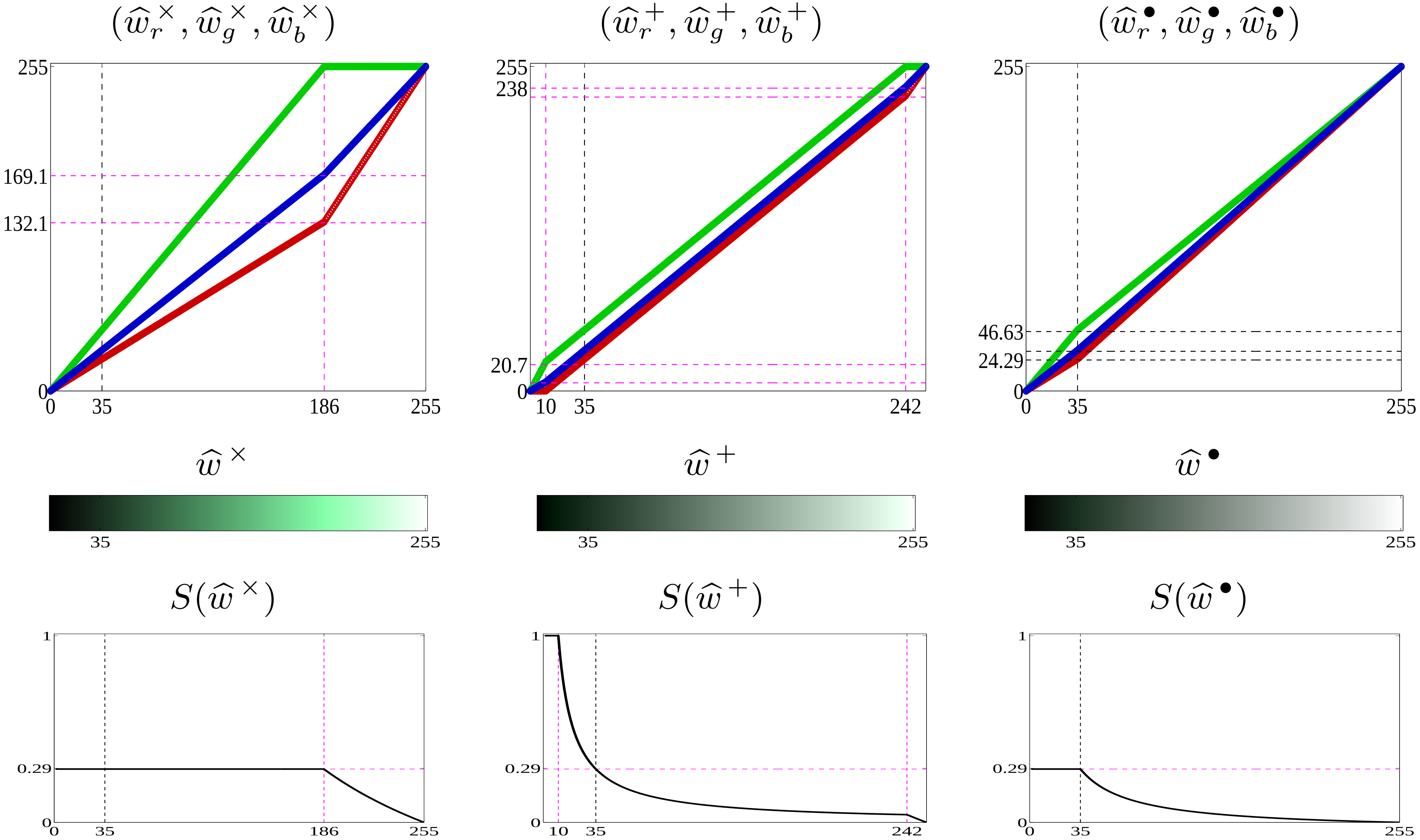
$$\begin{array}{lll} \text{(i)} & S(\widehat{w}^{\bullet}[i]) = S(w[i]) & \text{if} \quad i \in \mathbb{I}_n \setminus \mathcal{V}, \\ \text{(ii)} & S(\widehat{w}^{\bullet}[i]) = S(w[i]) \, \dfrac{f[i]}{\widehat{f}[i]} \, \dfrac{L-1-\widehat{f}[i]}{L-1-f[i]} & \text{if} \quad i \in \mathcal{V}. \end{array}$$

where $\mathcal{V} := \left\{ i \in \mathbb{I}_n \ : \ \widehat{f}[i] > f[i] \right\} \quad \Rightarrow \quad \mathcal{U}(1) \subseteq \mathcal{V}, \ \ \mathcal{L}(0) \subset \mathbb{I}_n \setminus \mathcal{V}$

Dark pixel

$$w = (25,\ 48,\ 32), \quad \text{intensity } f = 35, \quad H(w) = 137.3, \quad S(w) = 0.29$$

Algorithm $(\times)$ Algorithm $(+)$ Algorithm (NM)

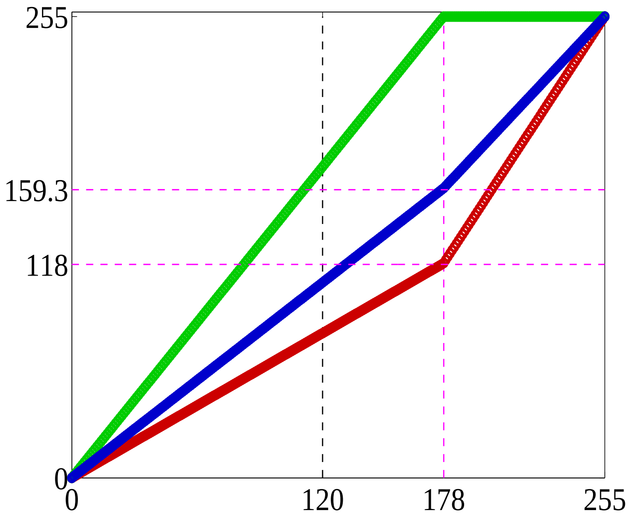


Bright pixel

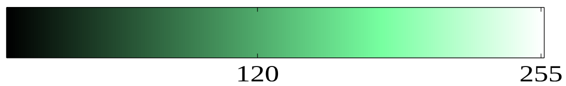
$$w = (80,\ 172,\ 108), \quad \text{intensity } f = 120, \quad H(w) = 137.3, \quad S(w) = 0.33$$

Algorithm $(\times)$

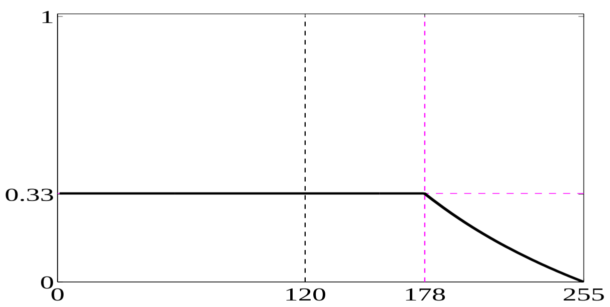
$(\widehat{w}_r^{\times}, \widehat{w}_g^{\times}, \widehat{w}_b^{\times})$



$\widehat{w}^{\times}$

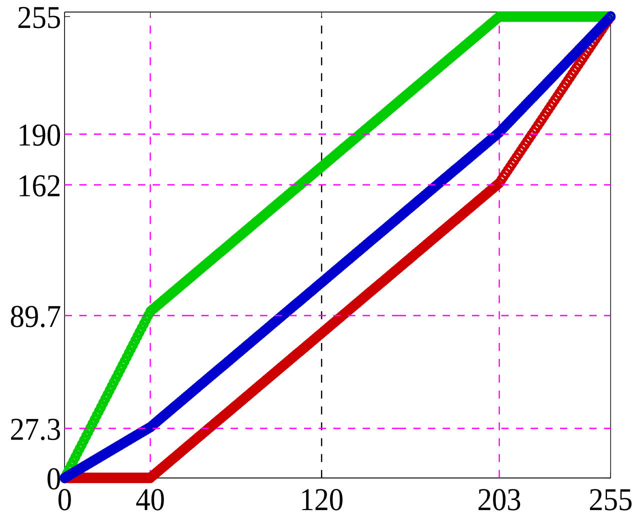


$S(\widehat{w}^{\times})$

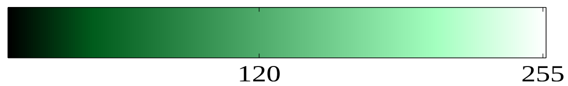


Algorithm $(+)$

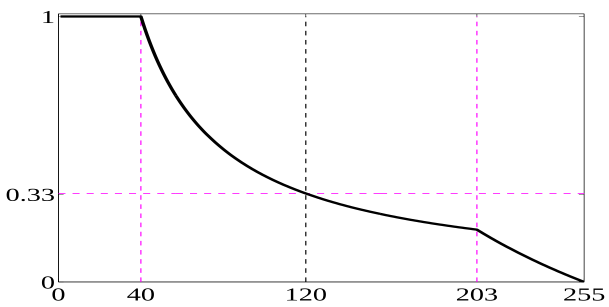
$(\widehat{w}_r^{+}, \widehat{w}_g^{+}, \widehat{w}_b^{+})$



$\widehat{w}^{+}$

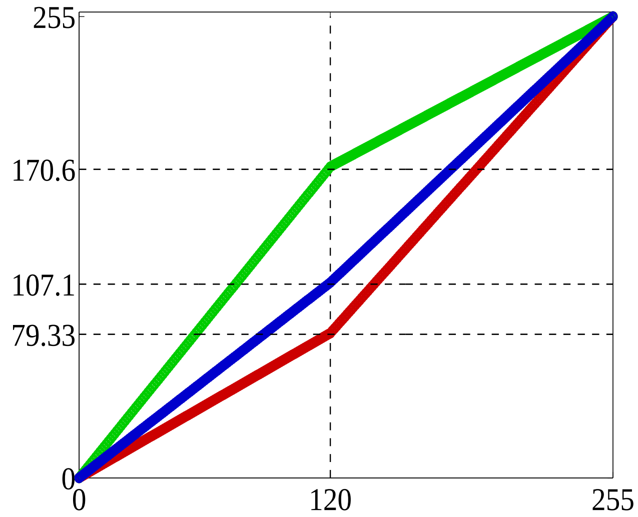


$S(\widehat{w}^{+})$

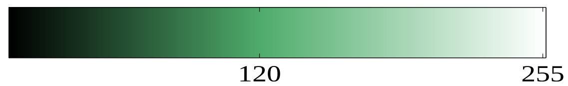


Algorithm (NM)

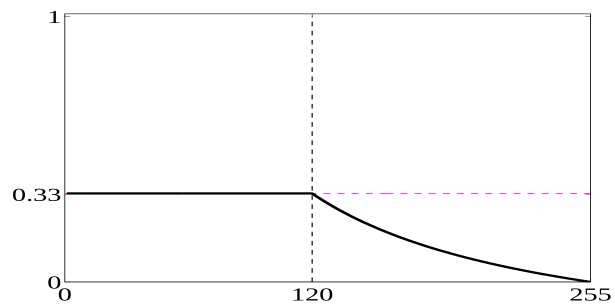
$(\widehat{w}_r^{\bullet}, \widehat{w}_g^{\bullet}, \widehat{w}_b^{\bullet})$



$\widehat{w}^{\bullet}$

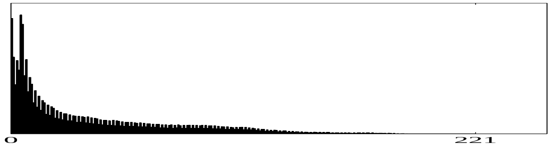


$S(\widehat{w}^{\bullet})$

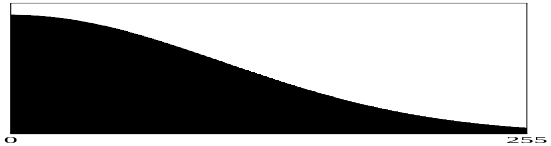


Case "dark pixel"

Input intensity histogram



Target histogram



Input image “bungalow” 660×1024

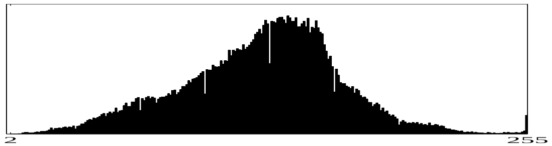
Algorithm $(\times)$

Alg. 5 $(+)$

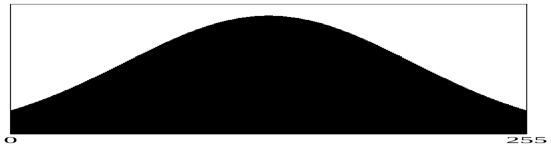
Alg. 2 (NM)

Case "bright pixel"

Input intensity histogram



Target histogram



Input image “flower” 660×1024

Algorithm ($\times$)

Algorithm ($+$)

Algorithm (NM)

5. Numerical Results

A. Target histograms

Well exposed images – bell-shaped histograms (Bovik 2005 and advices Photoshop)

Gaussian shaped target histograms $\widehat{h}_{\mathrm{G}}$ fixed so that

$$l := \widehat{h}_{\mathrm{G}}(0) \leq 1, \quad \max_{x \in [0, L-1]} \widehat{h}_{\mathrm{G}}(x) = 1 \text{ and } r := \widehat{h}_{\mathrm{G}}(L-1) < 1 \ .$$

User-free parameters:

- $l \in (0, 1]$ which is the desired portion of dark pixels in the enhanced image;
- $r \in (0, 1)$ drawing the desired portion of light pixels.

Compute $\mu := \ln l \, \frac{L-1}{\ln l - \ln r} \left(1 - \sqrt{\frac{\ln r}{\ln l}} \right)$ and $\sigma := -\frac{\mu^2}{\ln l}$. The *shape* of the target histogram is

$$\widehat{h}_{\mathrm{G}}(x) = \exp \left(-\frac{(x - \mu)^2}{\sigma} \right), \quad x \in [0, L-1] \ . \tag{1}$$

Normalization w.r.t. n (the number of pixels): $\widehat{h}(x) = \frac{n \, \widehat{h}_{\mathrm{G}}(x)}{\sum_{x=0}^{L-1} \widehat{h}_{\mathrm{G}}(x)} \quad \forall \, x \in \{0, \cdots, L-1\}$.

h_f - input intensity histogram (after stretching if no pixel values on $[0, L_0]$).

The choice of (l, r) to build $\widehat{h}_{\mathrm{G}}$ depends on h_f and the enhancement task.

E.g., $(l, r) = (1, 0.99)$ leads to HE. A too large r entails artifacts typical for HE.

i) $\widehat{h}_{\mathrm{G}}$ is easily adapted to all images with a roughly unimodal histogram

If the pixel values are mainly in the middle of $[0, L-1]$ and decay at the ends (flower, $\mathit{islanda}$, fields) choose $l \gtrapprox r \in [0.1,\ 0.2]$.

If the histogram rapidly decays towards $L-1$, choose $r \in (0, 0.2]$ ($\mathit{bungalow}$, $\mathit{boy\text{-}on\text{-}stones}$, $\mathit{cathedral}$, orchid, fields). The stronger this decay, the smaller the value of r (e.g. $\mathit{cathedral}$ $r = 10^{-4}$.)

If most of the pixel have small values (under-exposed images) select $l \in [0.8, 1]$. The higher the concentration near 0, the larger the value of $l \leq 1$n.

ii) For images with important very dark and very light areas, function $\widehat{h}_{\mathrm{G}}$ should not work well. Then a good option is to take a mixed target histogram

$$\widehat{h}_{\mathrm{mix}} := \frac{1}{2}(h_f + \widehat{h}_{\mathrm{G}})$$

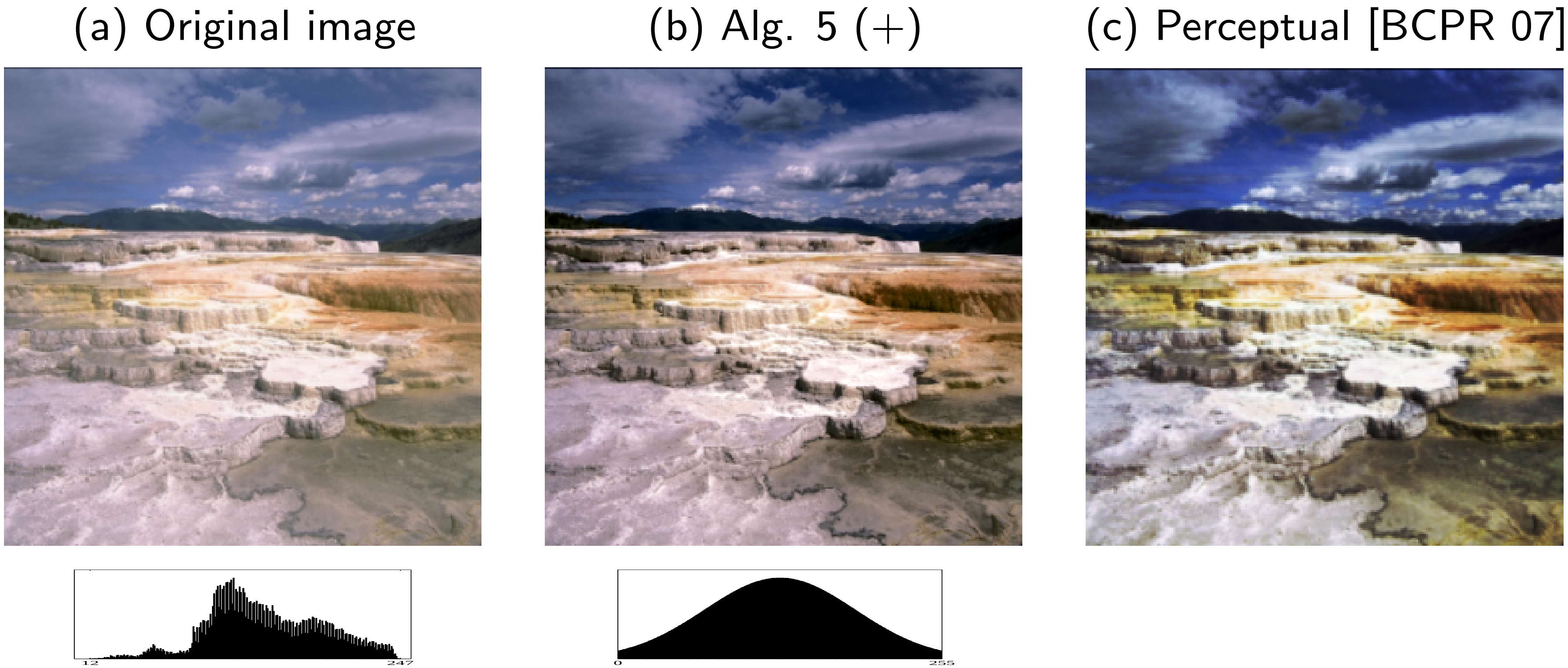
where (l, r) for $\widehat{h}_{\mathrm{G}}$ are selected following i). Examples: $\mathit{cathedral}$, $\mathit{Jericoacoara}$, $\mathit{ferrari}$.

Enhancement Examples

Most of the images there are photos that the authors could not take correctly.

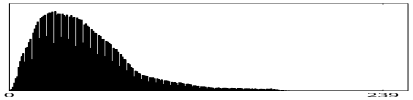
Comparison with

- Alg. 2 (NM) S. F. Naik and C. A. Murthy, “Hue-preserving color image enhancement without gamut problem”, *IEEE Trans. Image Process.*, vol. 12, no. 12, pp. 1591–1598, Dec. 2003.
- [BCPR 07] M. Bertalmío, V. Caselles, E. Provenzi, and A. Rizzi, “Perceptual color correction through variational techniques”, *IEEE Trans. Image Process.*, vol. 16, no. 4, pp. 1058–1072, Apr. 2007.
- [APBC 09] R. Palma-Amestoy, E. Provenzi, M. Bertalmío, and V. Caselles, “A perceptually inspired variational framework for color enhancement”, *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 31, no. 3, pp. 458–474, 2009.
- [ACE G 12] P. Gertreuer, “Automatic color enhancement (ACE) and its fast implementation”, *Image Processing On Line*, DOI: 10.5201/ipol.2012.g-ace, vol. 2012



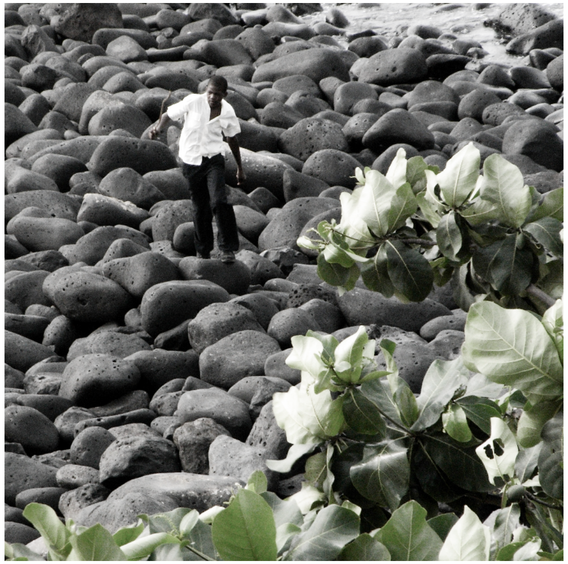
(a) Image *islanda* (courtesy of P. Greenspun) of size 294×293, and histogram of its intensity channel. Enhancement results: (b) Additive algorithm with Gaussian target histogram for $(l, r) = (0.1, 0.1)$; (c) Perceptual variational method by [BCPR 07] with Michelson's contrast function and default parameters (courtesy of the authors).

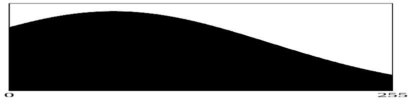
boy-on-stones (800×800)



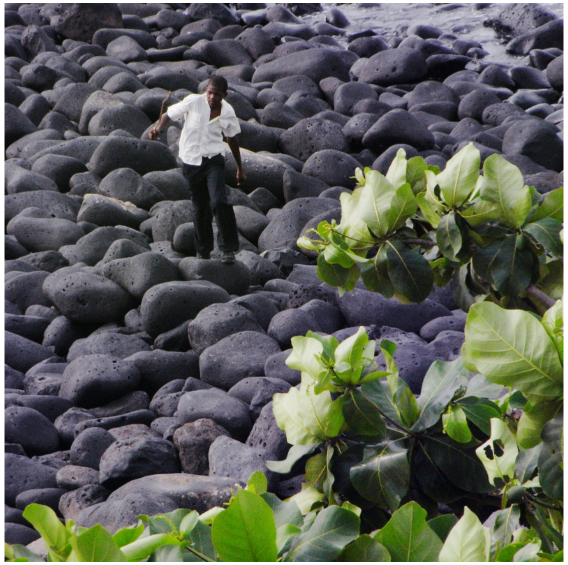
Alg. 4 $(\times)$ $(l, r) = (0.8, 0.2)$

Alg. 2 (NM)





Perceptual [BCPR 07]

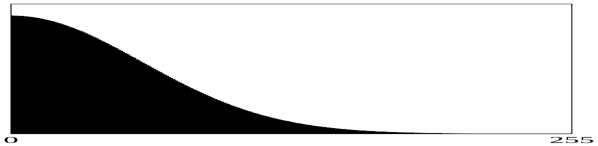


Perceptual [APBC 09]

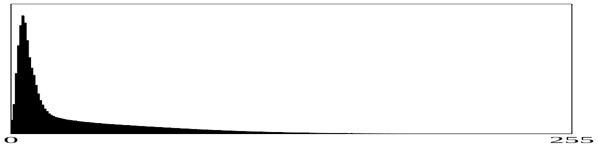
ACE [G 12]

“cathedral” 768×1024

Alg.4($\times$), $\widehat{h}_{\mathrm{G}}$ $(l,r) = (1, 10^{-4})$



Alg.4($\times$), $\widehat{h}_{\mathrm{mix}}$ $(l,r) = (1, 10^{-4})$



Perceptual [BCPR 07]

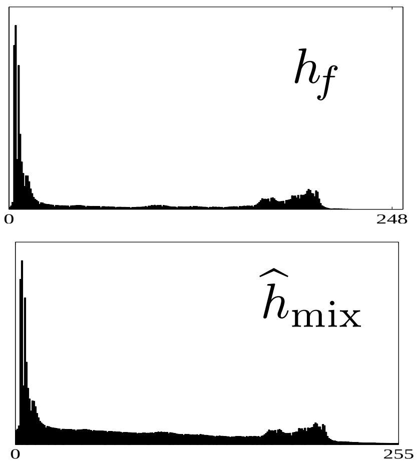
Perceptual [APBC 09]

Alg.2 (NM) $\widehat{h}_{\mathrm{G}}$

Jericoacoara (886×1181)

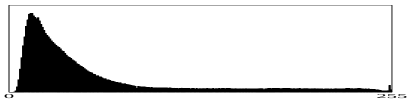
Alg. 5 $(+)$, $\widehat{h}_{\text{mix}}$ $(l, r) = (1, 0.1)$

Histograms

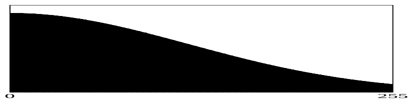


ACE [G 12] $\alpha = 3$

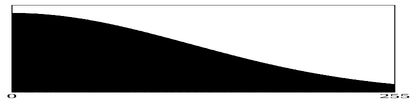
orchid (768×1024)



Alg.4 $(\times)$ $\widehat{h}_{\mathrm{G}}$ $(l, r) = (1, 0.1)$



Alg.5 $(+)$ $\widehat{h}_{\mathrm{G}}$



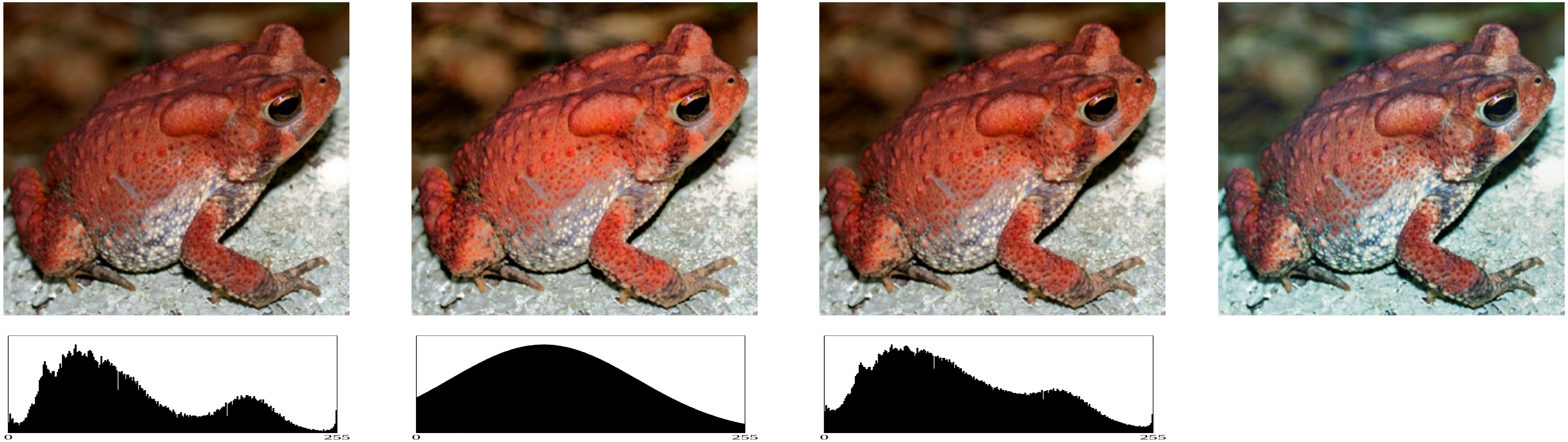
Alg. 2 (NM)

Perceptual [BCPR 07] $\gamma = 0.2$

ACE [G 12]

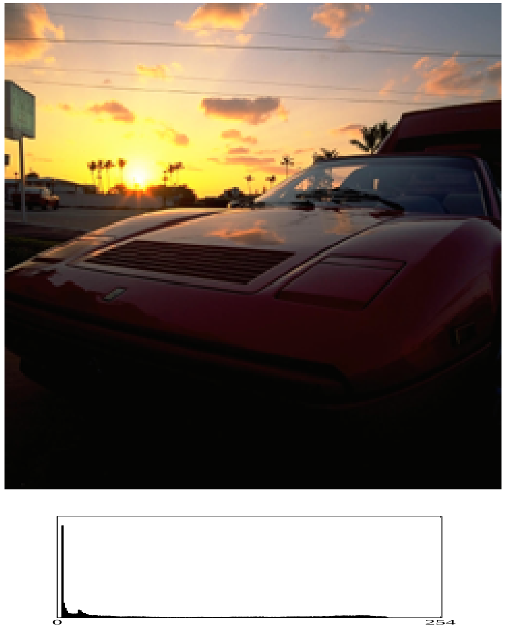
Input “orchid” with a bad flashlight effect.

(a) *frog* (332×300) | Alg.4 ($\times$) $\widehat{h}_{\mathrm{G}}$ $(0.4, 0.1)$ | Alg.4 ($\times$) $\widehat{h}_{\mathrm{mix}}$ | ACE [G 12] $\alpha = 3$

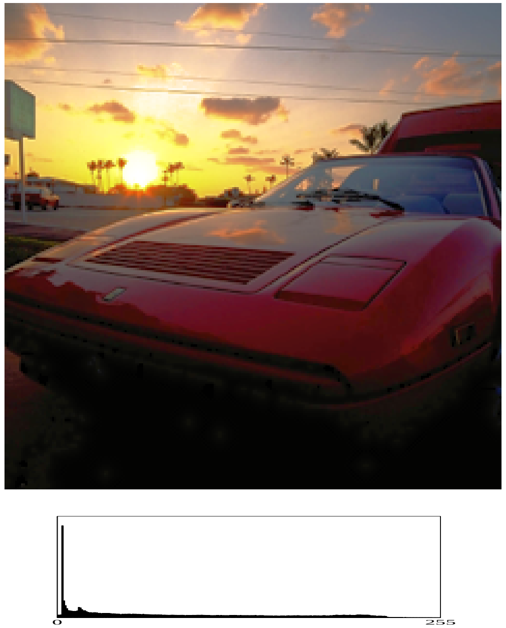


(a) Image credits: John D. Willson, USGS Amphibian Research and Monitoring Initiative)

(a) *ferrari* (235×240)



Alg. 4 $(\times)$, $\widehat{h}_{\mathrm{mix}}$ $(l, r) = (1, 0.1)$

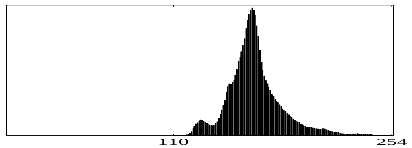


Perceptual [BCPR 07]

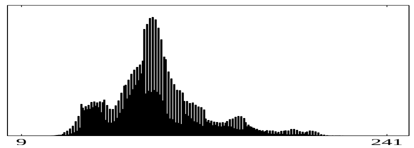
Perceptual [APBC 09]

(a) Ccourtesy of P. Greenspun. Perceptual methods: image credits to the authors.

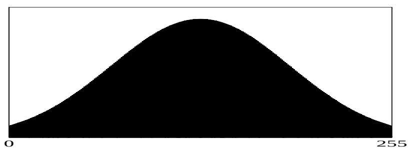
fields (512×512)



Stretch



Alg.4 $(\times)$ $\widehat{h}_{\mathrm{G}}$ $(l, r) = (0.1, 0.1)$



ACE [G 12] $\alpha = 8$

image	$\sharp\,\mathcal{U}(1)$	$\sharp\,\mathcal{U}(0)$	$\sharp\,\mathcal{L}(0)$	$\sharp\,\mathcal{V}$
bungalow $\widehat{h}_{\mathrm{G}}$	1.90	0.94	0	98.15
islanda $\widehat{h}_{\mathrm{G}}$	0.89	0.85	0.94	2.93
boy-on-stones $(\widehat{h}_{\mathrm{G}})$	4.76	2.40	0.54	94.15
cathedral $(\widehat{h}_{\mathrm{G}})$	1.04	0.00	1.38	95.71
Jericoacoara $(\widehat{h}_{\mathrm{mix}})$	5.98	5.71	0.40	68.26
orchid $(\widehat{h}_{\mathrm{G}})$	0.21	0.10	1.85	87.93
frog $(\widehat{h}_{\mathrm{G}})$	0.09	0.08	6.79	80.04
ferrari $(\widehat{h}_{\mathrm{mix}})$	4.40	4.22	0.30	60.7
fields $(\widehat{h}_{\mathrm{G}})$	4.61	2.99	2.06	89.16

Percentage of pixels requiring an upper or lower gamut correction in the HS based algorithms. For our Algorithms 4 ($\times$) and 5 ($+$) the numbers $\sharp\,\mathcal{U}(1)$, resp., $\sharp\,\mathcal{U}(0)$, $\sharp\,\mathcal{L}(0)$ are very small in all experiments which is not the case for $\sharp\,\mathcal{V}$ in the Naik-Murthy algorithm.

Summary and conclusions

To take home:

- color image enhancement algorithms consisting of two steps:
  - fast ordering algorithm and 1D histogram specification,
  - affine model with hue and optimal range preservation in RGB space
- important instances: additive and multiplicative algorithms
- simple and fast, does a good job
- flexible and stable w.r.t. the target histogram parameters
- analysis of the algorithms in terms of chromaticity improvement
- comparison with Naik-Murthy algorithm and with perceptional algorithms

Future research:

- target histograms according to needs and image content
- extension to video

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Theory and new algorithms for color image enhancement, HAL Preprint 2013.

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Thank you!